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AdS/CFT and gravity

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Abstract

The radiation-dominated $k = 0$ FRW cosmology emerges as the induced metric on a codimension one hypersurface of constant extrinsic curvature in the five-dimensional AdS-Schwarzschild solution. That we should get FRW cosmology in this way is an expected result from AdS/CFT in light of recent comments regarding the coupling of gravity to "boundary" conformal field theories. I remark on how this calculation bears on the understanding of Randall and Sundrum's "alternative to compactification." A generalization of the AdS/CFT prescription for computing Green's functions is suggested, and it is shown how gravity emerges from it with a strength $G_4 = 2G_5/L$. Some numerical bounds are set on the radius of curvature L of AdS_5. One of them comes from estimating the rate of leakage of visible sector energy into the CFT. That rate is connected via a unitarity relation to deviations from Newton's force law at short distances. The best bound on L obtained in this paper comes from a match to the parameters of string theory. It is $L \lesssim 1\,\mathrm{nm}$ if the string scale is $1\,\mathrm{GeV}$. Higher string scales imply a tighter bound on L.

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1 Introduction

The AdS/CFT correspondence [?, ?, ?] (see [?] for a review) relates a quantum field theory in one dimension to a theory in one higher dimension that includes gravity. The primary example is $\mathcal{N} = 4$ super-Yang-Mills theory, which is related to the dimensional reduction of type IIB string theory on S^5 to the five-dimensional non-compact geometry AdS_5 (anti de-Sitter space). The boundary of the Poincaré patch of AdS_5 is simply Minkowski space, except that the metric on the boundary is only specified up to a conformal transformation. This is OK because $\mathcal{N} = 4$ super-Yang-Mills theory is conformal, even at the quantum level. The correspondence is usually studied in a strong coupling region for the gauge theory, where it is far from classical, but the dual gravity picture is classical in the sense that curvatures are small on the Planck and string scales.

In [?] it was proposed that slices of AdS_5 could serve as an alternative to compactification manifolds. It was shown that when the near-boundary region of AdS_5 is cut away and the bulk spacetime simply ends on a wall of constant extrinsic curvature (a horosphere of AdS_5 to be precise), there is a normalizable graviton mode which has zero mass in the four dimensions of the boundary. The metric of AdS_5 is

$$ds_5^2 = e^{2r/L}(-dt^2 + d\vec{x}^2) + dr^2 \tag{1}$$

where $\vec{x}$ is an ordinary 3-vector. This metric is a solution to $R_{\mu\nu} = -\frac{4}{L^2} g_{\mu\nu}$. Hypersurfaces of constant r are horospheres. The part of the metric that is cut away in [?] is $r > r_*$ for some given r_*.

In fact the proposal of [?] was to glue two identical copies of the sliced anti-de Sitter space together along the $3+1$ dimensional boundary. However the four-dimensional graviton pointed out in [?] is quite a general phenomenon, and there is a large freedom in what one might have on the other side of the horospherical boundary of a given copy of AdS_5. An illustration of this can be found in [?], where a single copy of AdS_5 is obtained as part of a type IIB string compactification on an orientifold of T^6. The relevant point is that for a vacuum state of the theory, the extrinsic curvature of the boundary should be proportional to the induced metric:

$$\Theta_{ij} = -\frac{1}{L} g_{ij}^{(\mathrm{induced})} \,. \tag{2}$$

Regarding the boundary as a positive tension $3+1$-dimensional brane separating two sliced copies of AdS_5, this amounts to the statement that the stress energy of the brane should respect $3+1$-dimensional Poincaré invariance. The constant of proportionality, $1/L$, is required so that there is a balance between the brane tension and the bulk cosmological constant. More generally, a codimension one boundary of a five-dimensional space with no excitations on it should have the same property that $\Theta_{ij} = -\frac{1}{L} g_{ij}^{(\mathrm{induced})}$.

The authors of [?] termed their construction an "alternative to compactification," which seemed appropriate because one can travel an infinite spatial distance into the five-dimensional bulk. It should be remarked however that there is a length scale, L, involved in the construction: it is the scale of curvature of AdS_5.

It was suggested by J. Maldacena [?], and argued most convincingly by E. Witten [?] in response to [?, ?], that this "alternative" to compactification should properly be viewed in light of the AdS/CFT correspondence as a coupling of gravity to whatever strongly coupled conformal theory the AdS_5 geometry is dual to. From now on I will restrict myself to a minimal scenario where a single copy of AdS_5 is cut off by an end-of-the-universe brane. Such objects are well known in type I$'$ string theory [?] and in Horava-Witten theory [?], so there is no problem of principle in having a true end of the universe. However my comments basically apply to any compactification geometry which involves the near-horizon part of AdS_5.

The idea that the scenario of [?] is best viewed in the context of AdS/CFT has not been universally embraced, perhaps partly because it is hard to see what to do with it. (That difficulty is not usually regarded as fatal, but it does seem to have held up progress on the current issue). The goal of section 2 is to make the idea seem more definite by using it to derive the radiation-dominated Friedman-Robertson-Walker (FRW) cosmology. There have been several recent papers on a somewhat similar subject; these include [?, ?, ?], and the second of these has a much better list of references for brane-world cosmology than I can assemble. The approach is rather different from the older string cosmology literature; see [?] for references.

The approach of the current paper is distinctive in that an FRW solution can be obtained by altering properties of the bulk spacetime rather than by exciting matter on the cutoff brane. Readers determined to understand it from a brane-world perspective may find it most useful to think of the bulk as the background of near-extremal D3-branes. Although this is correct on the level of formulas, I consider the brane-world perspective subtly misleading: rather I would say that the entirety of the five-dimensional bulk is nothing more than a manifestation of the dynamics of a conformal field theory.

Section 3 consists of some remarks on the general framework of AdS/CFT with a cutoff brane. A generalization of the prescription [?, ?] for computing Green's functions is suggested at the level of effective field theory. The four-dimensional Einstein action can be derived in this formalism, with the result $G_4 = 2G_5/L$. This relation obtains regardless of the location of the cutoff brane in AdS_5. In standard AdS/CFT, where there is no cutoff brane and hence no normalizable graviton, the terms responsible for the Einstein action were removed using local counter-terms [?]. Corrections to Newton's force law are discussed, but I avoid presenting details since the idea of the calculation is not original to me.

Matter on the cutoff brane is incorporated naturally in the formalism. Although

I do not propose a definite model, the idea is to have visible sector matter on the cutoff brane, somewhat as in certain heterotic M-theory models [?]. Excitations of that matter would have to dominate over the solution in section 2 for $z \lesssim 10^{10}$ in order for the cosmology to be realistic.

In section 4, I make some rough numerical estimates. One is to check the cosmological effects of visible sector matter losing its energy to the CFT. In AdS language this corresponds to absorption of bulk gravitons by the horizon of the Poincaré patch, as in [?, ?, ?]. The rate of energy loss is directly related to the deviations from Newton's force law. Another is to estimate parameters of string theory that would permit the deviations from Newton's force law to be observed experimentally, assuming that AdS_5 emerges from a string theory construction. To obtain deviations at the scale of even a nanometer (still three orders of magnitude below the sensitivity of proposed experiments) an extremely low string scale is required—approximately $1\,\mathrm{GeV}$. We make some speculative remarks regarding low string scales.

Throughout the paper, μ will denote a five-dimensional bulk spacetime index and i is a four-dimensional index. In cases where precision is required, I will denote five-dimensional coordinates as $x^\mu = (t, \vec{x}, r)$ and four-dimensional coordinates as $\xi^i = (\tau, \vec{\xi})$. Throughout the paper, ℓ_{Pl} will denote the four-dimensional Planck length: $\ell_{\mathrm{Pl}} = \sqrt{G_4} \approx 1.6 \times 10^{-33}\,\mathrm{cm}$ in units where $\hbar = c = 1$.

2 A "cosmological" solution

Let us start purely from a four-dimensional point of view, and turn on a finite temperature for the conformal field theory which is small in Planck units. If we calculate the corresponding energy density ρ, use the trivial equation of state $p = \rho/3$, and apply the standard equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\rho\,, \tag{3}$$

what must result is the standard radiation-dominated FRW cosmology,

$$ds^2 = -d\tau^2 + a(\tau)^2 d\vec{x}^2\,, \tag{4}$$

where $a(\tau)^2$ is linear in τ.[1] I will always use τ for four-dimensional cosmological time; t will be reserved for the Poincaré time in AdS_5. The only difficulty is finding the constant of proportionality in $\rho \propto T^4$: if the conformal field theory is interacting, it could be a non-trivial exercise even in the limit $T \ll 1/\ell_{\mathrm{Pl}}$ where gravity loops shouldn't matter. However if the theory in question has an AdS dual, then the study of black

[1] There is probably no obstacle in principle to extending the calculation to positive spatial curvature. However for negative spatial curvature I worry that the pathology observed in [?] might emerge.

holes in AdS guarantees the relation

$$\rho = \frac{3\pi^2}{2} c T^4 \,, \tag{5}$$

where c is the coefficient for the trace anomaly in a normalization where $c = (N^2 - 1)/4$ for $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills theory.[2] In this normalization, a single Abelian photon has $c = 1/10$. There is a standard relation in AdS/CFT [?]

$$c = \frac{\pi}{8} \frac{L^3}{G_5} \,. \tag{6}$$

It should be remarked that for $\mathcal{N} = 4$ gauge theory at weak coupling, (5) becomes $\rho = 2\pi^2 c T^4$. This is the 4/3 problem, first noted in [?, ?], and now understood as being a result of the strong interactions.

So far we have employed the AdS/CFT correspondence merely as a tool for determining a detail of the strong-coupling thermodynamics. However the calculation can be done entirely on the AdS side if we take seriously the idea that the cutoff brane is no more nor less than a coupling of gravity to the conformal field theory. It seems inevitable from a string theory perspective that the gravity would be quantized, and that the detailed "structure" of the cutoff brane encodes the details of the quantum gravity; but by taking $T \ll 1/\ell_{\rm Pl}$ we will be able to ignore this issue. The cutoff brane, or "Planck brane," controls gravity, while the bulk of AdS_5 controls the conformal field theory. By assumption, the Planck brane is not appreciably influenced by finite temperature, but the conformal field theory is; so we should retain (2), but change the bulk background from AdS_5 to AdS_5-Schwarzschild. The metric of AdS_5-Schwarzschild is

$$\begin{aligned} ds_5^2 &= e^{2r/L} \left(-h(r) dt^2 + d\vec{x}^2 \right) + \frac{dr^2}{h(r)} \\ h(r) &= 1 - b^4 e^{-4r/L} \\ b &= \pi L T_0 \,. \end{aligned} \tag{7}$$

Here T_0 is the Hawking temperature associated with the time coordinate t. It is a constant parameter of the AdS_5-Schwarzschild solution. The calculation will deal only with the coordinate patch covered by $(t, \vec{x}, r)$.

Given an orientable surface with unit normal n_i (which specifies a notion of outside and inside by the direction in which it points), the extrinsic curvature can be defined as $\Theta_{ij} = -(\delta_i^k - n_i n^k) \nabla_k n_j$. We will follow [?] in taking n_i to be the outward unit

[2] The calculation of [?] shows that in the limit where classical gravity is applicable to the AdS black holes there is in fact only one independent coefficient in the trace anomaly. This and (5) are non-trivial constraints on theories which can have AdS duals.

normal, which points toward the true boundary of AdS_5. There is a set of solutions to (2) which form a foliation of the coordinate patch in question:

$$\frac{t}{L} = \frac{e^{r/L}}{b^2} + \frac{1}{4b} \sum_{k=1}^{4} i^k \log(1 - i^k b e^{-r/L}) + \frac{t_0}{L} \,, \tag{8}$$

where t_0 is a constant of integration specifying a particular leaf. All leaves have the same induced metric since they are related by translation in t. Using (2) with the same constant of proportionality, $1/L$, avoids a four-dimensional cosmological constant—more about this later. It proves most convenient to parametrize a particular leaf by $(r_*, \vec{x})$, where $r = r_*$ is a solution of (8) for r in terms of t. Then the induced metric is

$$ds^2_{\text{(induced)}} = -\frac{e^{4r_*/L}}{b^4} dr_*^2 + e^{2r_*/L} d\vec{x}^2 \,. \tag{9}$$

If we define

$$\tau = L \frac{e^{2r_*/L}}{2b^2} \tag{10}$$

then the metric (9) assumes the standard FRW form:

$$\begin{aligned} ds^2_{\text{(induced)}} &= -d\tau^2 + a(\tau)^2 d\vec{x}^2 \\ a(\tau) &= b\sqrt{\frac{2\tau}{L}} \,. \end{aligned} \tag{11}$$

Thus we do indeed observe the linear $a(\tau)^2$ that we expected. This behavior is strictly a consequence of conformal invariance: any conformal field theory provides a source term for Einstein's equations just like a bunch of massless photons.

We can be a little more quantitative and rederive the coefficient in (5) from the new perspective. In the late time limit, we are safe using the relation

$$\frac{1}{G_4} = \frac{1}{G_5} \int_{-\infty}^{r_*} dr \, e^{2(r - r_*)/L} = \frac{L}{2G_5} \,. \tag{12}$$

Actually this relation comes from a Kaluza-Klein reduction of five-dimensional gravity to four for a horospherical Planck brane in pure AdS_5. It should be OK to leading order for a brane in an asymptotically AdS_5 region of bulk spacetime, provided the brane is only slightly curved on the scale L. Such a brane is locally like a horosphere of AdS_5. It is perhaps more common in the literature to see (12) quoted without the $e^{-2r_*/L}$ in the denominator. The discrepancy is merely due to the fact that our four-dimensional metric is precisely the induced metric, whereas more commonly the four-dimensional metric on a horosphere is taken to be $e^{-2r_*/L} ds^2_{\text{(induced)}}$. The form of (12) is forced on us by the choice $ds_4^2 = ds^2_{\text{(induced)}}$, which does seem the natural one in the present context.

Combining the relation

$$\frac{1}{4\tau^2} = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\rho \tag{13}$$

with (12) and (6) leads to

$$\rho = \frac{3b^4}{16\pi G_5 L e^{4r_*/L}} = \frac{3\pi^2}{2} c(e^{-r_*/L} T_0)^4 \,. \tag{14}$$

Now, the temperature T_0 measured with respect to the time t is not the same as the temperature T measured with respect to the time τ; rather,

$$T = \frac{dt}{d\tau} T_0 = e^{-r_*/L} T_0 \,, \tag{15}$$

where we have used the relation $t = \sqrt{2L\tau}/b$. So (14) is indeed identical to (5), coefficient and all.

The reader who feels that the calculation of section 2 must be some sort of cheat is reminded that *String theory on AdS_5 is identical to a 3+1-dimensional conformal field theory.* This aphorism is not intended to increase the sense of puzzlement, but rather to resolve it, as follows. We wanted our cosmology to be driven by the conformal field theory dual to the bulk AdS geometry rather than by anything on the Planck brane. So we left the Planck brane in its ground state and made the bulk AdS geometry thermal by adding a black hole horizon.

It is worth remarking that I employed no stabilization mechanism because none was needed. From a brane-world point of view, the worst has already happened: the negative tension brane of [**?**] has retreated to infinity,[3] and the delicate near-horizon cusp has been cut off by a finite temperature horizon. The effect of that horizon is most transparent when viewed in light of the AdS/CFT correspondence: it means that the $3 + 1$-dimensional conformal field theory is at finite temperature.

In a generic bulk geometry, the retreating Planck brane would cause the four-dimensional Newton constant to change. In this regard, an asymptotically AdS space is very special: provided we use the induced metric on the Planck brane (rather than some warping of it) as the four-dimensional metric, (12) will hold asymptotically when the Planck brane is moving in the asymptotically AdS region with curvatures which are small compared to L.

That $ds^2_{\text{(induced)}}$ turned out to be *exactly* the radiation-dominated FRW metric should excite some suspicion on the part of the reader. Mightn't there be quantum gravity effects at sufficiently early times which modify the picture? We derived the agreement between (5) and (14) using a late-time relation, (12). It seems like a massive

[3]The reality of such a negative tension object is something I am only provisionally willing to allow for the sake of argument, since I am aware of no fully satisfactory string theory construction of it in an AdS background.

conspiracy that the physics of early times would arrange for the cosmology to remain exactly radiation-dominated FRW. Unfortunately I cannot say much about that since the details of gravity are tied up at least in part in the nature of the Planck brane. One correction that should be within reach is the effect of the conformal anomaly.

Is this real cosmology? Not as it stands: nucleosynthesis would be dramatically spoiled if the "hidden CFT" that AdS_5 represents had any sizable effect on the radiation-dominated era of our universe. However it is straightforward to extend the discussion by adding matter to the brane, and its stress tensor, $T_{ij}^{(\text{matter})}$, could take over from T_{ij}^{CFT} at late times. (If all we're worried about is nucleosynthesis, then late times means $z \lesssim 10^{10}$). The AdS/CFT equation relevant to such phenomena is

$$\begin{aligned} G_{ij}^{(\text{induced})} - 8\pi G_4 T_{ij}^{(\text{CFT})} - 8\pi G_4 T_{ij}^{(\text{matter})} = \\ -\frac{2}{L}\left[\Theta_{ij} - \left(\Theta + \frac{3}{L}\right) g_{ij}^{(\text{induced})}\right] - 8\pi G_4 T_{ij}^{(\text{matter})} . \end{aligned} \tag{16}$$

This equation is a rearrangement of a formula obtained in [?] in the course of deriving quasi-local stress-energy tensors for AdS/CFT in various dimensions.[4] I have used (12) to define G_4. It is necessary to check that visible matter does not lose energy to the CFT fast enough to spoil the cosmology. Since the CFT's couplings are essentially of gravitational origin, this is perhaps plausible. An estimate will be presented in section 4.

Actually, (16) is content-free from a physical perspective: it is only AdS/CFT's statement of translation from purely four-dimensional quantities to quantities which constrain how the Planck brane sits in the five-dimensional spacetime. All the physics comes from setting either side equal to zero. In this section we considered a case where $T_{ij}^{(\text{brane})} = 0$; then setting the right hand side equal to zero amounts to requiring (2). Solving (2) gave us back radiation-dominated FRW cosmology, which perhaps sounded surprising; but the identity (16) makes it inevitable, because if the right hand side vanishes, so must the left hand side.

Suppose now we knew all about the matter on the brane, and discovered that it generated a positive cosmological constant: $-8\pi G_4 T_{ij}^{(\text{induced})} = \Lambda g_{ij}^{(\text{induced})}$ with $\Lambda > 0$. Assuming the AdS part to be at zero temperature, we would then reduce (16) to

$$G_{ij}^{(\text{induced})} + \Lambda g_{ij}^{(\text{induced})} = -\frac{2}{L}\left[\Theta_{ij} - \left(\Theta + \frac{3}{L} + \frac{L^2\Lambda}{2}\right) g_{ij}^{(\text{induced})}\right] . \tag{17}$$

It is straightforward to show that setting the right hand side equal to zero leads to an induced metric which is dS_4. Approximately this calculation has appeared elsewhere in the literature ([?, ?] are the citations which I am aware of in five bulk dimensions; also

[4] Only I have changed the sign of the Einstein tensor. This is necessary because of a difference in sign conventions. The derivation in section 3 will serve as a check that the signs in (16) are consistent.

there is a direct analog in lower dimensions [?]. This last reference has a fairly general discussion of homogenous isotropic induced metrics on codimension one domain walls in AdS_4.)

3 The general framework

The equation (16) is an approximate first variation of a more general relation, which is the natural extension of the prescriptions of [?, ?]:

$$\begin{aligned} S_{\rm eff}[\gamma_{ij},\psi] &= S_{\rm 4d\,gravity}[\gamma_{ij}] + S_{\rm 4d\,matter}[\gamma_{ij},\psi] + W_{\rm CFT}[\gamma_{ij}] \\ &= \underset{g_{ij}\to\gamma_{ij}}{\rm extremum}\left(S_{\rm bulk}[g_{\mu\nu}] + S_{\rm brane}[\gamma_{ij},\psi]\right) . \end{aligned} \tag{18}$$

The first equation indicates a natural way of splitting up the four-dimensional effective action into four-dimensional gravity, the CFT, and the four-dimensional matter which comes from excitations on the Planck brane. The second equation is the actual statement of AdS/CFT, which in this case includes a "brane reduction" of five-dimensional gravity to four dimensions, as envisaged in [?, ?]. $W_{\rm CFT}$ is the generating functional of connected Green's functions of the conformal field theory, γ_{ij} is the metric on the Planck brane, and ψ are the extra matter fields which live on the Planck brane. The extremum on the right hand side is taken subject to the boundary condition that the metric induced from $g_{\mu\nu}$ on the cutoff brane is γ_{ij}. It is the saddle-point approximation to quantum gravity in the bulk. If we wanted to do quantum gravity in some more complete way (e.g. string theory), we would make the replacement extremum $S \to \frac{1}{i}\log\int[\mathcal{D}g]e^{iS}$, where $\int[\mathcal{D}g]$ represents path integration. (Path integration in the sense that I mean is ill-defined in our current understanding of string theory, but we might imagine some equivalent way of improving the saddle point approximation). In a real string theory model, there would probably be many more bulk fields besides the metric $g_{\mu\nu}$ that $S_{\rm bulk}$ would depend on, and they would also have to have their boundary values specified in the extremum (or path integral). $W_{\rm CFT}$ would depend on those boundary values, and there would also be new terms added to $S_{\rm 4d\,gravity}$ for the dynamics of the zero modes of the extra fields. How much of a problem all this extra junk is depends on the couplings to the standard model fields. The optimistic view is that such couplings are about as important in particle physics contexts as the coupling of electrons and quarks to gravity. The zero modes of extra bulk fields would modify long-distance four-dimensional gravity if they remained massless, but any sort of confinement or mass generation mechanism could prevent this problem.

Like all of AdS/CFT, (18) is a claim to be substantiated rather than an assumption. However, no proof of it can be complete because $W_{\rm CFT}$ is a complicated non-local functional of γ_{ij} whose exact form would be independently accessible to us only through a strong coupling QFT computation. If one takes the boundary to be the true boundary of AdS_5, the evidence is compelling [?] that the extremum on the right hand side of (18) does indeed lead to the generating functional of connected Green's functions for a CFT. Through the UV-IR relation we understand that cutting off a portion of AdS_5 should change physics in the ultraviolet only. Thus (18) is true insofar as it is well-defined (that is, on the level of an effective field theory on scales much lower than the four-dimensional Planck scale) provided we can show that $S_{\rm 4d\,gravity} + S_{\rm 4d\,matter}$ emerges from the extremum on the right hand side. That is what I will actually demonstrate concretely. In the process I will derive (12) in a general setting, and also check that the sign that seemed worrisome in (16) is OK.

The proof is piggy-backed on the calculations of [?]. In order to keep the presentation self-contained, I will recapitulate parts of that work. The setting is a foliation of a five-dimensional Einstein manifold $\mathcal{M}$ (for instance, AdS_5 or AdS_5-Schwarzschild), whose boundary has a metric in the conformal class of a specified metric $g_{(0)}$, and whose metric can be written in the form [?]

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{4\rho^2} d\rho^2 + \frac{1}{\rho} \bar{g}_{ij} d\xi^i d\xi^j \,. \tag{19}$$

In (19) and the following equations, L has been set to 1. The metric $\bar{g}_{ij}$ can depend on ρ, but according to [?, ?] it has an expansion

$$\bar{g} = \bar{g}_{(0)} + \rho \bar{g}_{(2)} + \rho^2 \log\rho\, \bar{h}_{(4)} + \rho^2 \bar{g}_{(4)} + \ldots \,. \tag{20}$$

Here $\bar{g}_{(2)}$ and $\bar{h}_{(4)}$ are tensors constructed from $\bar{g}_{(0)}$ using with two and four derivatives, respectively. The expansion breaks down after the logarithmic term, in the sense that the $\bar{g}_{(n)}$ are no longer covariant tensors. Fortunately the first two terms of (20) are all that we will need. Explicitly,

$$\bar{g}^{(2)}_{ij} = \frac{1}{2} \left(\overset{o}{R}_{ij} - \frac{1}{6} \overset{o}{R}\, \bar{g}^{(0)}_{ij} \right) , \tag{21}$$

where $\overset{o}{R}_{ij}$ is the Ricci tensor of $\bar{g}^{(0)}_{ij}$ and $\overset{o}{R}$ is the associated Ricci scalar.

The action under the extremum in (18) is

$$S_{\rm bulk}[g_{\mu\nu}] + S_{\rm brane}[\gamma_{ij}, \psi] = \frac{1}{16\pi G_5} \int_{\mathcal{M}} d^5x\, \sqrt{g}\, [R + 20] + \frac{1}{16\pi G_5} \int_{\partial\mathcal{M}_\epsilon} d^4\xi\, \sqrt{\gamma}\, [-2\Theta + \alpha] \,. \tag{22}$$

We have located the cutoff brane on the hypersurface $\partial\mathcal{M}_\epsilon$ defined by the equation $\rho = \epsilon$. The tensor γ_{ij} is the induced metric on the cutoff brane, and

$$\alpha = \alpha_0 + \frac{16\pi G_5}{\sqrt{\gamma}} \mathcal{L}_{\rm matter}(\gamma_{ij}, \psi) \,. \tag{23}$$

The constant α_0 is what we will adjust to balance the tension of the Planck brane against the bulk cosmological constant. An imperfect adjustment would lead to the dS_4 induced metric, as commented on after (17). Thus we are not claiming to make headway on the cosmological constant problem; rather, we are pushing it into the Planck brane. The extrinsic curvature term in (22) is necessary in order to have a well-defined variational principle.

Extremizing (22) with the induced metric γ_{ij} held fixed can be achieved by letting ds_5^2 have the form (19) (this is at least true up to errors which will be subleading in a derivative expansion). Then (cf. (10) of [?])

$$\underset{g_{ij}\to\gamma_{ij}}{\text{extremum}} \left(S_{\text{bulk}}[g_{\mu\nu}] + S_{\text{brane}}[\gamma_{ij},\psi]\right) = \frac{1}{16\pi G_5}\int d^4\xi\,\mathcal{L} \tag{24}$$

where

$$\begin{aligned}\mathcal{L} &= 4\int_\epsilon \frac{d\rho}{\rho^3}\sqrt{|\det \bar{g}|} + \left[\frac{1}{\rho^2}(-8+4\rho\partial_\rho+\alpha)\sqrt{\bar{g}}\right]_{\rho=\epsilon} \\ &= \sqrt{|\det \bar{g}_{(0)}|}\left[\frac{\alpha-6}{\epsilon^2} + \frac{\alpha}{2\epsilon}\,\text{tr}\,\bar{g}_0^{-1}\bar{g}_2 - \log\epsilon\,\mathring{a}_{(4)} + (\text{finite})\right].\end{aligned} \tag{25}$$

Here we have defined

$$\mathring{a}_{(4)} = -\frac{1}{8}\mathring{R}^{ij}\mathring{R}_{ij} + \frac{1}{24}\mathring{R}^2\,. \tag{26}$$

This quantity was identified in [?] as the conformal anomaly of the CFT. The AdS/CFT prescription as detailed there is simply to remove the terms that diverge as $\epsilon\to 0$ via local counterterms. This is the only sensible course if the ultimate goal is to take $\epsilon\to 0$ so that the cutoff boundary becomes the true boundary. Instead we want to keep the cutoff boundary at a finite, arbitrary ϵ and regard the induced metric γ_{ij} on $\partial\mathcal{M}_\epsilon$ as the Einstein metric of the four-dimensional world. Rewriting (25) in terms of γ_{ij}, one finds

$$\mathcal{L} = \sqrt{|\det\gamma|}\left[\alpha - 6 + \frac{1}{2}R - \log\epsilon\, a_4 + \ldots\right], \tag{27}$$

where now R is the Ricci scalar of the metric γ_{ij} and a_4 is defined as in (26) only using curvature tensors pertaining to γ_{ij} rather than to $\bar{g}_{ij}^{(0)}$. One might fear that the logarithmic term in (20) would contribute to the $\log\epsilon$ term in (27). It doesn't because $\text{tr}\,\bar{g}_{(0)}^{-1}\bar{h}_{(4)} = 0$.

Because powers of ϵ cancel in (27) (and ϵ is finite anyway) there is no longer an expansion parameter in (27). The expansion can only be justified as a derivative expansion, provided that the embedding of the cutoff brane in the five-dimensional Einstein space involves only curvatures which are slight on a length scale L. Combining (23) and (24) with (27), setting $\alpha_0 = 6$, and repristinating powers of L, we find

$$\begin{aligned}&\underset{g_{ij}\to\gamma_{ij}}{\text{extremum}} \left(S_{\text{bulk}}[g_{\mu\nu}] + S_{\text{brane}}[\gamma_{ij},\psi]\right) = \\ &\qquad \frac{L}{32\pi G_5}\int d^4\xi\,\sqrt{\gamma}R + \int d^4\xi\,\sqrt{\gamma}\mathcal{L}_{\text{matter}}(\gamma_{ij},\psi) + W_{\text{CFT}}[\gamma_{ij}]\end{aligned} \tag{28}$$

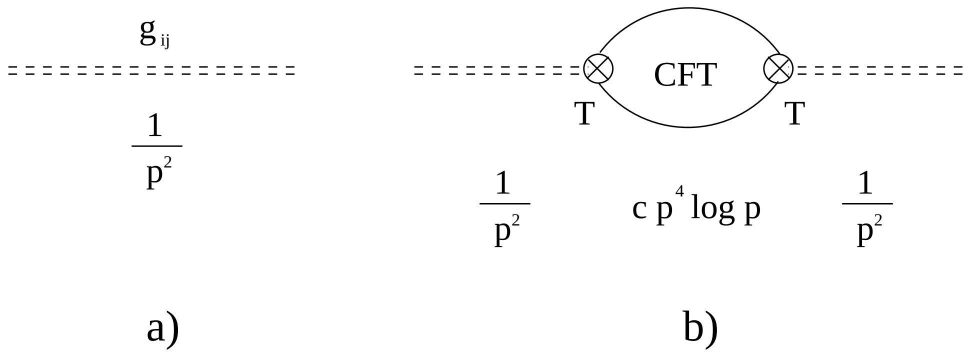

Figure 1: Contributions to the graviton propagator, following [?]: a) free graviton propagation; b) leading CFT correction. The blob between the stress tensor insertions is intended to denote the full $\langle TT\rangle_{\rm CFT}$ correlator.

where $W_{\rm CFT}$ includes the $\log\epsilon$ term in (27) plus all the other terms which we indicated with $\ldots$. We indeed verify the relation $G_4 = 2G_5/L$. Also, since the Ricci scalar came in with the right sign in (28), the signs of (16) are consistent. The calculation leading to (28) is similar to Kaluza-Klein reduction, the main difference being that the relation $G_4 = 2G_5/L$ does not involve the total length of the fifth dimension (which could be infinite), but rather the curvature scale of the five-dimensional geometry. This makes the current scenario rather different from those of [?], where the circumference of the extra dimensions does affect the four-dimensional Planck length.

Extremizing (22) with respect to the metric without restricting the induced metric on the cutoff brane would amount to setting the right hand side of (16) to zero. The trick of (18) is to extremize first with the induced metric held fixed and then argue (as we are in the process of doing) that the extremization that remains to be carried out gives us the equations of gravity (and brane matter if we want it), plus something non-local which I called $W_{\rm CFT}$. The claim that this something arises equivalently by integrating out a CFT is the substance of AdS/CFT and the basis for the suggestions in [?, ?].

The argument (19)-(28) stands in relation to the observation [?] of a normalizable graviton approximately as the derivation of the low-energy effective action of string theory via beta-functions stands in relation to the calculation of the massless string spectrum.

With enough terms computed in the derivative expansion of $W_{\rm CFT}[\gamma_{ij}]$, it should be possible to obtain corrections to Einstein's equations and hence Newton's force law to any desired order. In fact it seems likely that the leading correction is already be accessible from the anomaly term a_4. However it is more transparent to follow the analysis of [?], where we merely differentiate $W_{\rm CFT}[\gamma_{ij}]$ twice with respect to γ_{ij} to obtain the first correction to the graviton propagator (see figure 1). The position space two-point function of the CFT stress tensor has the form $\langle T(x)T(0)\rangle \sim c/x^8$. In momentum space this is $\langle T(p)T(-p)\rangle \sim c\,p^4 \log p$. The corrected graviton propagator is

$$\begin{aligned} G^{(2)}(p) &\sim \frac{1}{p^2} + \frac{1}{p^2}\ell_{\rm Pl}\left(c\,p^4\log p\right)\ell_{\rm Pl}\frac{1}{p^2} \\ G^{(2)}(x) &\sim \frac{1}{x^2} + c\,\ell_{\rm Pl}^2\frac{\log x}{x^4}\,, \end{aligned} \tag{29}$$

where the factors of the four-dimensional Planck length are vertex factors for the coupling of the stress tensor to the graviton. The altered propagator gives rise to deviations from Newton's $1/r^2$ force law:

$$F = \frac{Gm_1m_2}{r^2}\left(1 + a_1\frac{L^2\log(a_2 r/\ell_{\rm Pl})}{r^2} + \ldots\right)\,, \tag{30}$$

where a_1 and a_2 are dimensionless numbers, and a_1 is on the order of unity. In (30) I have used (6) and (12) to combine $c\,\ell_{\rm Pl}^2$ into $\pi L^2/4$. I do not claim any originality for the computation in (29). The only remark I would add to the recorded comments in [?] is that the coefficient of the logarithmic term is indeed computable: up to factors of order unity it is G_4 times the central charge of the CFT.

Clearly, by differentiating (18) and keeping track of all the Lorentz structure we could obtain the corrected propagator in complete detail and extract the exact value of a_1. I will refrain from entering into this computation here because another group is pursuing similar lines [?]. It was important however to present the general outline of the analysis because it will figure prominently in the next section. Before closing I would remark that the constant a_2 seems to me a rather non-universal quantity, likely to depend on the structure of what we have called the Planck brane. I have no argument why $\ell_{\rm Pl}/a_2$ should or should not actually be on the order of L itself. The only thing one can say for sure about the experimental signature is that it involves $\log r$ and becomes visible on the length scale L.

4 Preliminary estimates

Note that (6) and (12) together imply that the central charge is $c = \frac{\pi}{4}\frac{L^2}{\ell_{\rm Pl}^2}$, where as usual $\ell_{\rm Pl}$ is the four-dimensional Planck length. For AdS_5 backgrounds arising from type IIB geometries including D3-branes, $c \sim N^2$ where N is the number of D3-branes. So $N \sim L/\ell_{\rm Pl}$. To be definite, let us suppose that L is on the order of a micron. Measurements of gravity in the foreseeable future might probe Newton's force law to this short a distance. From current measurements we can already say $L \lesssim 1\,{\rm mm}$ (modulo the uncertainty of a_1, which I have not computed). $L \sim 1\,\mu{\rm m}$ means $N \sim 10^{29}$. This number seems on the high side for a string compactification:

something has to soak up the five-form flux. D3-brane charge is conserved, so it is true that if we managed to set $N = 10^{29}$ through some arcane string theory construction, we wouldn't worry about it wiggling. As disciples of AdS/CFT we would also be relieved that five-dimensional quantum gravity effects aren't an immediate problem. However, in order not to spoil nucleosynthesis, we should start by requiring that $\rho_{\rm CFT} \ll \rho_{\rm CBR}$, where $\rho_{\rm CBR}$ is the energy density of the cosmic background radiation. That translates roughly to $T_{\rm CFT} \lesssim T_{\rm CBR}/c^{1/4}$, or $T_{\rm CFT} \lesssim 10^{-14}\,{\rm K}$ today if we want L on the order of a micron.[5] To summarize,

$$c \sim N^2\,, \qquad L \sim N\ell_{\rm Pl}\,, \qquad T_{\rm CFT} \lesssim T_{\rm CBR}/\sqrt{N}\,. \tag{31}$$

Suppose the CFT is cold enough to satisfy $\rho_{\rm CFT} \ll \rho_{\rm CBR}$.[6] Cosmology could still be spoiled if energy leaks too quickly from visible matter into the CFT. The analogous problem in theories with compact extra dimensions is cooling by emission of bulk gravitons [**?**]. To evaluate whether there is a problem in our case, we must investigate the mechanisms of thermal equilibration between the CFT and the other matter in the universe, operating on the assumption that the CFT is very cold. Fortunately the tools are already partly in hand. Standard Model particles can lose energy to the conformal field theory through processes controlled by the graph in figure 2a). The inclusive rates from these graphs are related to the logarithmic correction to Newton's law through the unitarity relation illustrated in figure 2b). In particular, the inclusive rate goes as $\ell_{\rm Pl}^2 L^2$. By dimensional analysis the contribution they make to the loss of Standard Model energy density over time is

$$\left(\frac{d\rho}{d\tau}\right)_{\rm lost} = a_3 \ell_{\rm Pl}^2 L^2 T_{\rm SM}^9\,, \tag{32}$$

where a_3 is a dimensionless number of order unity and $T_{\rm SM}$ is the temperature of Standard Model excitations. From an AdS point of view, (32) is literally the rate at which energy density falls across the horizon to be absorbed by the D3-branes. Three powers of $T_{\rm SM}$ come from the absorption cross section [**?**, **?**, **?**]; also there are powers of $T_{\rm SM}$ from the finite temperature kinematics of the Standard Model particles. Energy density also decreases because of Hubble expansion: in total,

$$\frac{d\rho_{\rm SM}}{d\tau} = -\frac{\dot a}{a} T_{\rm SM}^4 - \ell_{\rm Pl}^2 L^2 T_{\rm SM}^9\,, \tag{33}$$

[5] It has long surprised me that AdS/CFT predicts $\rho \sim cT^4$: naively counting flat directions in $\mathcal{N} = 4$ super-Yang-Mills theory suggests $\rho \sim \sqrt{c}T^4$. Even if this were somehow true for a special CFT, it would only soften (31) to $T_{\rm CFT} \lesssim T_{\rm CBR}/N^{1/4}$.

[6] Section 2 treated the opposite limit. One should be able to use the equation (16) with $T_{ij}^{CFT} = 0$ to find a hypersurface in AdS_5 whose induced metric is real-world cosmology. But this is only an equivalent means to find what we already know by solving Einstein's equations. So we will "cast down the ladder" and work directly in four dimensions whenever possible.

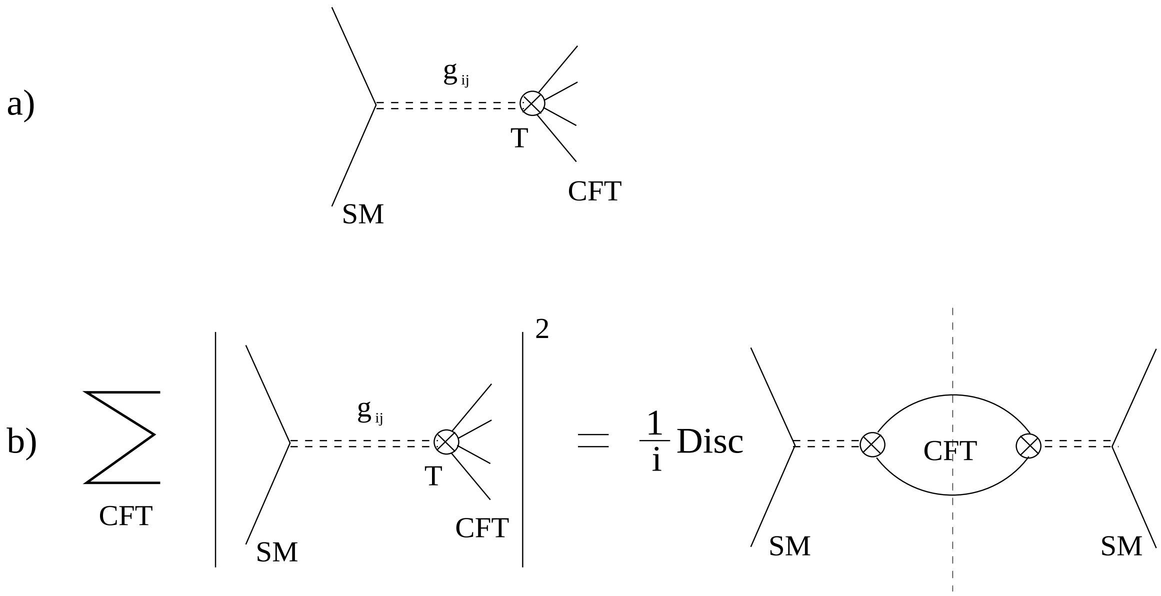

Figure 2: a) Standard Model particles losing energy via graviton exchange to CFT excitations. b) The inclusive rate is given by a unitarity cut of the first correction to the graviton propagator.

where we have dropped factors of order unity. One such factor is the central charge of the Standard Model fields which are light compared to the temperature at any given time. Approximately this same factor appears in both terms on the right hand side of (33), so it doesn't matter much for the relative size of the terms. However a_3 does matter, and so do the factors of 2 and π from the Stefan-Boltzmann constant that we have suppressed in the first factor. I have also suppressed a term in (33) for CFT energy leaking back into visible fields, but that is OK since we are operating on the assumption that the CFT is cold.

To determine whether the CFT is appreciably affecting the cosmology, one should compare the two terms in (33). Their ratio is

$$\kappa = \ell_{\rm Pl}^2 L^2 T_{\rm SM}^5 H^{-1} \tag{34}$$

where $H^{-1} = a/\dot{a}$ is the inverse Hubble time (a function of τ). The CFT will not appreciably affect cosmology as long as $\rho_{\rm CFT} \ll \rho_{\rm SM}$ and κ is small. What small means in this context depends on all the "factors of order unity" that we have dropped. All these factors are calculable: once we have (6) and (12) the rest is essentially kinematics. In order to make some preliminary estimates I will assume that the Hubble expansion term in (33) dominates over the energy loss term when $\kappa \ll 1$.

The state of the universe today does not lead to a dramatic bound on L: one finds

$$L^2 \ll \frac{1}{\ell_{\rm Pl} H_o^{-1} T_{\rm CBR}^5} \sim 10^{33}\,{\rm cm}^2\,, \tag{35}$$

which is easily passed by any realistic theory. However the bound tightens as one goes back in time. Tracing the matter-dominated cosmology back to the time of last scatter at $z \sim 10^4$, one obtains roughly

$$L^2 \ll 10^{33}\,{\rm cm}^2 \left(\frac{a_{\rm last\ scatter}}{a_o} \right)^5 \frac{H_o^{-1}}{H_{\rm last\ scatter}^{-1}} = 10^{33}\,{\rm cm}^2 \left(\frac{a_{\rm last\ scatter}}{a_o} \right)^{7/2} = 10^{19}\,{\rm cm}^2\,, \tag{36}$$

still not meaningfully restrictive. Tracing the radiation-dominated cosmology back to nucleosynthesis at $z \sim 10^{10}$, one obtains (by an estimation which now may be excessively naive)

$$L^2 \ll 10^{19}\,{\rm cm}^2 \left(\frac{a_{\rm nucleosynthesis}}{a_{\rm last\ scatter}} \right)^5 \frac{H_{\rm last\ scatter}^{-1}}{H_{\rm nucleosynthesis}^{-1}} = 10^{21}\,{\rm cm}^2 \left(\frac{a_{\rm nucleosynthesis}}{a_{\rm last\ scatter}} \right)^3 = 10\,{\rm cm}^2\,. \tag{37}$$

Still this bound is satisfied with four orders of magnitude to spare (in L) if we suppose L to be on the order of a micron. I emphasize the extreme simple-mindedness of the estimates: all I have done in (37) is to write

$$L^2 = \kappa \frac{1}{\ell_{\rm Pl}^2 T_{\rm SM}^5 H^{-1}} \approx \kappa \frac{1}{\ell_{\rm Pl}^2 T_{\rm CBR}^5 H_o^{-1}} \frac{1}{z_{\rm last\ scatter}^{7/2}} \left(\frac{z_{\rm last\ scatter}}{z_{\rm nucleosynthesis}} \right)^3\,, \tag{38}$$

and then demand $\kappa \ll 1$. The powers of z in (36)-(38) arise from the relations $H^{-1} \sim a^{3/2}$ for the matter dominated cosmology and $H^{-1} \sim a^2$ for the radiation dominated cosmology. In view of the actual number obtained in (37), a more accurate estimate would be desirable. One can also attempt to trace cosmology back to larger z and tighten the bound on L further, if one feels convinced that κ must still be small for $z > 10^{10}$.

An independent bound on L could obtained by checking the effect on supernovas of energy loss to the CFT, as in [?]. The energy scales here are on the order of 30 MeV, so a slightly better bound than (37) might be expected.

There is yet another way to set a bound on L if we assume that the AdS_5 geometry comes from type IIB string theory through some Freund-Rubin ansatz or related compactification. In such compactifications, the extra five dimensions have the same length scale L as AdS_5. Suppose $AdS_5 \times S^5$ is the relevant geometry. Then the standard string theory relation $16\pi G_{10} = (2\pi)^7 g_s^2 \alpha'^4$ combined with (12) and $\mathrm{Vol}\, S^5 = \pi^3 L^5$ leads us to

$$L = g_s^{1/3} \left(\frac{16\pi^3 \alpha'^4}{G_4} \right)^{1/6}\,. \tag{39}$$

Type IIB theory has an S-duality symmetry which takes $g_s \to 1/g_s$. Thus we can assume that $g_s \leq 1$. A conventional value of $\sqrt{\alpha'}$ would be only a few times the four-dimensional Planck length, $\ell_{\rm Pl}$. This results in a bound on L which is also a few times $\ell_{\rm Pl}$. In order to make L observably big, we would have to make $\sqrt{\alpha'}$ big too. What is the biggest $\sqrt{\alpha'}$ we could possibly imagine? In the old days of string theory the answer would have been $\sqrt{\alpha'} \sim 1\,{\rm GeV}^{-1} \approx 0.2\,{\rm fm}$: this is literally the Regge slope of observed hadronic spectra. In recent literature [?], values $\sqrt{\alpha'}$ as big as $1{\rm TeV}^{-1}$ have been regarded as acceptable. Plugging these numbers into (39) leads to

$$\begin{aligned} L &\lesssim 10^{-7}\,{\rm cm} \qquad \text{for } \sqrt{\alpha'} \sim 1{\rm GeV}^{-1} \\ L &\lesssim 10^{-11}\,{\rm cm} \qquad \text{for } \sqrt{\alpha'} \sim 1{\rm TeV}^{-1}\,. \end{aligned} \tag{40}$$

The bad news is that deviations from Newton's force law on length scales this small wouldn't detected any time soon. The good news is that standard cosmology is no problem, as far back as nucleosynthesis and further. If we assume that the radiation-dominated FRW solution still pertains, we can estimate the redshift z_* and the thermal energies T_* at which $\kappa = 1$. The result is

$$\begin{aligned} z_* &\sim 10^{15}\,, \quad T_* \sim 100\,{\rm GeV} \qquad \text{for } \sqrt{\alpha'} \sim 1{\rm GeV}^{-1} \\ z_* &\sim 10^{18}\,, \quad T_* \sim 100\,{\rm TeV} \qquad \text{for } \sqrt{\alpha'} \sim 1{\rm TeV}^{-1}\,. \end{aligned} \tag{41}$$

To get the numbers in (41) we have combined several approximations and assumptions. The "error in the exponent" should probably be taken to be about ± 2. It is somewhat suggestive that the values of T_* we found are "within errors" of the boundary of our direct knowledge of particle physics. If the string scale is at a TeV, then physics changes sufficiently there that we can no longer have any confidence that the radiation-dominated FRW cosmology is relevant. Thus the second line of (41) only shows that there are no cosmological problems as far back as we can trace the theory. Strings at a GeV are a different matter, and we will return to them shortly.

Although type IIB string theory provides the best-understood vacua involving AdS_5, it is conceivable that some other type of string theory, even a non-critical string, could have an AdS_5 vacuum: see for example [?]. For a non-critical string, (39) would not be the right estimate, since some or all of the five compact dimensions simply aren't there. Suppose the non-critical string lives in n dimensions, with $n \geq 5$. Assume also that it exhibits some form of S-duality, so that the coupling cannot be parametrically large. Then

$$L \lesssim \left(\frac{\sqrt{\alpha'}}{\ell_{\rm Pl}} \right)^\gamma \sqrt{\alpha'} \tag{42}$$

where $\gamma = 2/(n-4)$. If we allow n to range from 10 to 5, the corresponding range of γ is from 1/3 to 2. It is also conceivable that some intersecting configuration of branes in critical string theory could have an AdS_5 component in its near-horizon geometry, and a different relation from (39) could pertain if some of the branes had more than $3+1$ worldvolume dimensions. I am not currently aware of any well-defined non-critical string theory other than the $c \leq 1$ toys. Nor can I give a definite example of intersecting branes with an AdS_5 near-horizon geometry. Besides, if the extra dimensions of the intersecting branes are larger than L, then the salient physics of extra dimensions would be more along the lines of [?] than [?]. For the sake of a concrete discussion, let us stick to (39), with (42) as a possible alternative.

Once we have ventured to set $\sqrt{\alpha'} \approx 1\,\mathrm{GeV}^{-1}$, the burning question is why all collider physics from a GeV up to a TeV isn't dramatically different. The simplest answer is rather iconoclastic. It is that from a four-dimensional point of view, strings are nothing more than QCD flux tubes. For energies well above $1\,\mathrm{GeV}$, a better set of variables is the quarks and gluons of the Standard Model. In the low-energy regime where strings are the good variables, we would quantize them and find a massless graviton. There could also be a massless open string spectrum if the Planck brane involves D-branes; such states would have to be included in the visible sector. Intuitively, the reason why a disk diagram with two gluon boundary insertions and one bulk insertion of a graviton wouldn't couple gluons to gravity on the scale of femtometers is that the wavefunction overlap is small. This is the magic of extra dimensions (exploited similarly in [?]): (39) is roughly a condition on how big the extra dimensions have to be in the well-understood type IIB string theory examples to make low-energy strings consistent with gravity at the four-dimensional Planck scale. The modified relation (42) could be pertinent for alternative models, as discussed in the previous paragraph.

The view taken in the previous paragraph is distinct from those of [?] or [?]. Wilson loops in AdS/CFT usually seek out a location of large redshift in the bulk geometry in order to lower their tension to the scale of confinement. The current scenario has Wilson loops terminating on the Planck brane, and the relevant geometry is the geometry near the Planck brane. I would not exclude scenarios where a large redshift does exist near the Planck brane, and the parameter entering into the Regge relation is a redshifted α'. If that is the way we think QCD strings are realized, then once again the bound on L is tighter than $L \lesssim 1\,\mathrm{nm}$, since the α' that enters (39) is the honest-to-goodness, un-redshifted string tension.

Strings at a GeV seem like a natural apotheosis of the proposals of [?, ?]. We do not have to "get rid" of the graviton if there are extra dimensions on the scale of a nanometer. (Significantly smaller L would work in a model where (42) pertains). We do not have to worry about nucleosynthesis if the estimate (37) bears out. But we do have to face some hard questions. First, if $L \sim 1\,\mathrm{nm}$, how do we manage to accommodate $N = L/\ell_{\mathrm{Pl}} \approx 10^{26}$ D3-branes? Something has to soak up all the Ramond-Ramond flux, and that sounds like an impossible stretch for string compactifications (see for example

[?]). We should remember however that either small g_s or models where (42) pertains could lower N. Second, string theory would have to face up to hadron physics in the energy range between pions and partons. Regge trajectories are as suggestive as they always were, but there is much more to be explained. Processes where some or all of the final energy winds up in CFT excitations could easily be a problem. However the relevant branching ratios depend on L rather than α', and amount to yet another way of setting an upper bound on L. Third, strings could stretch from the Planck brane all the way into the AdS_5 bulk (to connect with a D3-brane if one wants to think in those terms) at only a finite cost in energy. The mass of such a string is roughly L/α', which comes out to be approximately $3000\,\mathrm{TeV}$ if we use $L \sim 1\,\mathrm{nm}$. This is out of the range of colliders, but it is nevertheless a dangerous number for any sort of loop computation because these strings are so numerous: there are as many of them as there are D3-branes. Again (42) alleviates the difficulty somewhat: $L/\alpha' \sim 10^{18\gamma}\,\mathrm{GeV}$. Fourth and finally, if flux tubes are long strings ending on the Planck brane, then what are quarks?

There is a precise way of characterizing the strings stretched from the Planck brane to the horizon of AdS_5: they are the massive W bosons associated with the separation of the Planck brane from the L/ℓ_{Pl} D3-branes that create the AdS_5 geometry. The scale of this Higgsing could be lowered, say to $30\,\mathrm{TeV}$, if L falls sufficiently short of saturating the bound in the first line of (40). Or we could return to strings at a TeV and get approximately the same $30\,\mathrm{TeV}$ Higgs scale by saturating the bound in the second line of (40). Either way, we are left with a version of [?], only with an enormous hidden sector gauge group and strings at a GeV or a TeV. In [?], it seemed like coupling the CFT to gravity might resurrect the hierarchy problem. This is less of a problem if the string scale is smaller than or comparable to the scale of soft breakings of the CFT, since gravity is a low-energy phenomenon. One may then inquire whether there is any reason to have an ultraviolet cutoff for the CFT. In AdS language, the Planck brane could just be a test brane moving in a AdS_5, with no part of it cut away. Now there is no obstacle to having a large number of D3-branes, since AdS_5 is non-compact. But the existence of a normalizable four-dimensional graviton is an issue for the consistency of the picture.

In conclusion, insisting that AdS_5 has to come from string theory provides a bound on L which is sharper than we were able to obtain from nucleosynthesis, and which appears to rule out experimental observation of (30). There are two reasons why string theory demands a small L. First, $L/\ell_{\mathrm{Pl}} \sim N$, where N is the number of units of Ramond-Ramond five-form flux. It is hard to make this number really big in string compactifications. Second, $G_4 L^6 \lesssim \alpha'^4$, so we can only get big L if we allow big α'. In an attempt to be maximally optimistic about the size of L, we have reconsidered strings at a GeV. Even this radical step only gave us $L \lesssim 1\,\mathrm{nm}$. If we made α' even bigger, it would only heightens the difficulties we encountered trying to make sense of GeV strings. The strategies proposed in AdS/CFT contexts to relate strings to QCD flux tubes generally have the property that the fundamental α' is smaller than $1\,\mathrm{GeV}^{-2}$, implying a tighter bound on L.

5 Discussion

The FRW cosmology found in section 2 is an interesting check of the claim that the "alternative to compactification" proposed in [?] is simply a conformal field theory coupled to four-dimensional gravity. However, as emphasized in section 4, the CFT shouldn't make any sizeable contribution to the actual cosmology of our universe at times later than $z = 10^{10}$. Before that time, one is entitled to speculate about the physical relevance of the solution of section 2. Suppose that the CFT and the visible sector matter on the Planck brane were in thermal equilibrium at some early time. Assuming that the CFT has a much larger central charge, we have $\rho_{\mathrm{CFT}} \gg \rho_{\mathrm{matter}}$, and the solution found in section 2 should approximately describe the cosmology. At late times one needs $\rho_{\mathrm{CFT}} \ll \rho_{\mathrm{matter}}$. In parallel with [?], we might imagine an inflationary scenario where the inflaton lives on the Planck brane. Then reheating directly affects only the visible sector, and if κ is small by the time of reheating there is substantially no thermal equilibration with the CFT.

In a scenario with GeV strings, thermalization with the CFT sets in significantly around $100\,\mathrm{GeV}$ (although we must recall that the estimates here were extremely crude). That alone might lead us to rule this case out unless a reheating mechanism could be proposed at a lower scale.

In known string compactifications, the number N of D3-branes is typically on the order of 10. As many as 10^3 D3-branes were claimed to be attainable in certain orbifold examples [?]. If we take this as a strict bound, then the relation $L/\ell_{\mathrm{Pl}} \sim N$ puts our entire discussion at an inaccessibly small length scale: $L \sim 10^{-30}\,\mathrm{cm}$ for $N = 10^3$. (Recall, ℓ_{Pl} is the four-dimensional Planck scale). The formalism developed in section 3 could still be useful for extracting a "low-energy" effective theory—"low-energy" being interpreted now as much less than $10^{16}\,\mathrm{GeV}$. Standard inflation occurs around $10^{14}\,\mathrm{GeV}$, so it is possible one might embed a "conventional" inflationary model in AdS using the dS_4 solution discussed after (17). The amusing aspect of such a model is that there is a natural candidate for the pre-inflationary universe: it is the radiation-dominated FRW solution found in section 2.

There are two solid conclusions to be drawn from the estimates of section 4. First, our present understanding of nucleosynthesis would not be threatened if deviations from Newton's force law of the form (30) were found. We already know that such deviations cannot be present on scales much larger than a millimeter, and this is enough to suppress the associated loss of energy to the conformal field theory for z as large as 10^{10}. Second, string theory as we understand it seems to forbid an AdS_5 space large enough to cause measurable deviations from Newton's force law. Even if we are willing to take the string scale down to $1\,\mathrm{GeV}$ and regard strings as collective effects of QCD, L still can't be larger than $1\,\mathrm{nm}$.

There is nothing sacred about an AdS_5 bulk spacetime: it has been the focus of so much recent literature in part because it is simple. Practically any string theory realization of AdS_5 will include scalar fields, and if they have a non-trivial profile, large deviations from AdS_5 are the generic behavior. The literature on renormalization group in AdS/CFT flows provides ample evidence of this (see for example [?, ?, ?, ?, ?, ?]—this is far from an exhaustive list). Only a subset of these geometries can support finite temperature, due to boundary conditions on the scalars at the black hole horizon. A felicitous feature of AdS_5, which will not be shared by generic "RG flow" geometries, is that the relation $G_4 = 2G_5/L$ obtains no matter where the Planck brane is in the bulk geometry. The formalism worked out in section 3 will still retain its general features in a more generic bulk geometry, but details will be rather different: for instance, it is no longer clear that the induced metric on the cutoff brane will be the Einstein frame metric.

A cutoff brane in a bulk geometry whose AdS/CFT dual is a quantum field theory undergoing renormalization group flow corresponds to gravity coupled to that same QFT. We might attempt to put the entire Standard Model in the depths of bulk, far from the Planck brane; this would be more in the spirit of [?]. Such a construction would be interesting, but AdS/CFT would give us little computational power over it: in almost all the examples I know, the "Standard Model" branes would sit at a region of strong curvatures. The exceptions are compactifications of type I$'$ string theory or Horava-Witten theory, where negative tension "ends of the world" are possible. It is possible to develop a formalism similar to the one in section 3 for these constructions, but it would have more the flavor of an ordinary Kaluza-Klein reduction, where heavy fields are integrated out and light fields are kept. The distinctive feature of (18) is that it enables us to integrate out *infrared* degrees of freedom.

String theory and string dualities have taught us that extra dimensions are theoretically inexpensive. But the view of the fifth dimension espoused in the current paper is not excessively literal: rather than making the claim that there is actually a large extra dimension of space waiting to be discovered, the statement is that an extra dimension is a convenient way to describe collective phenomena of a strongly coupled quantum field theory—in the present case, a conformal field theory coupled to gravity. To make this seem more definite, suppose measurements of gravity at a micron did after all turn up deviations from Newton's law of the form (30). The "AdS" interpretation would be that gravitons are propagating in the fifth dimension, while the "CFT" interpretation would be that a loop of gauge bosons in a purely four-dimensional theory had contributed. Which interpretation we prefer is a matter of ontology: if AdS/CFT is right then they are absolutely indistinguishable on experimental grounds. I confess that my current ontology isn't very happy either with a CFT with $c \sim 10^{58}$ or with a fifth dimension with curvatures on the scale of a micron. But it is in the subtle guises of string duality and string compactification that I suspect extra dimensions have the best chance of improving our understanding of the physical world.

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